Space and Time: From Antiquity to Einstein and Beyond

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At the beginning of the 20th century, Einstein revolutionized the notions of space and time, first through special relativity and then, a decade later, through general relativity. Conceptual ideas underlying general relativity are explained and its physical ramifications summarized in general terms, without recourse to advanced mathematics. This theory is perhaps the most sublime creation of the human mind. Nonetheless, it has become increasingly clear that it too has serious limitations which can be overcome only through another dramatic revision of our notions of space and time. The article concludes by providing glimpses of what awaits us in the 21st century.

I. FROM ANTIQUITY TO EINSTEIN

"As an older friend, I must advise you against it, for, in the first place you will not succeed, and even if you succeed, no one will believe you."
Max Planck to Albert Einstein, on learning that Einstein was attempting to find a new theory of gravity to resolve the conflict between special relativity and Newtonian gravity (1913).

Every civilization has been fascinated by notions of Space (the Heavens) and Time (the Beginning, the Change and the End). Early thinkers from Gautama Buddha and Lao Tsu to Aristotle commented extensively on the subject. Over centuries, the essence of these commentaries crystallized in human consciousness, providing us with mental images that we commonly use. We think of space as a three dimensional continuum which envelops us. We think of time as flowing serenely, all by itself, unaffected by forces in the physical universe. Together, they provide a stage on which the drama of interactions unfolds. The actors are everything else —stars and planets, radiation and matter, you and me.

For over a thousand years, the four books Aristotle wrote on physics provided the foundation for natural sciences in the Western world. While Heraclitus had held that the universe is in perpetual evolution and everything flowed without beginning or end, Parmenides had taught that movement is incompatible with Being which is One, continuous and eternal. Aristotle incorporated both these ideas in his 'cosmogonic system. Change was now associated with the earth and the moon because of imperfections. Changelessness was found on

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other planets, the sun and stars because they are perfect, immutable and eternal. In modern terms one can say that in Aristotle's paradigm, there was absolute time, absolute space *and* an absolute rest frame, provided by earth. This was the reigning world-view Isaac Newton was exposed to as a student at Cambridge in the years 1661-65.

Twenty years later, Newton toppled this centuries old dogma. Through his Principia, first published in 1686, he provided a new paradigm. Time was sill represented by a 1-dimensional continuum and was absolute, the same for all observers. All simultaneous events constituted the 3-dimensional spatial continuum. But there was no absolute rest frame. Thanks to the lessons learned from Copernicus, earth was removed from its hitherto privileged status. Galilean relativity was made mathematically precise and all inertial observers were put on the same physical footing. The Principia also shattered Aristotelian orthodoxy by abolishing the distinction between heaven and earth. The heavens were no longer immutable. For the first time, there were universal principles. An apple falling on earth and the planets orbiting around the sun were now subject to the same laws. Heavens were no longer so mysterious, no longer beyond the grasp of the human mind. Already in the beginning of the 1700s, papers began to appear in the Proceedings of the Royal Society, predicting not only the motion of Jupiter but even of its moons! No wonder then that Newton was regarded with incredulity and awe not only among lay people but even among leading European intellectuals. For example, Marquis de l'Hôpital —well known to the students of calculus for the l'Hôpital rule— eagerly wrote from France to John Arbuthnot in England about the Principia and Newton: Good god! What fund of knowledge there is in that book? Does he eat & drink & sleep? Is he like other men? As Richard Westfall put it in his authoritative biography of Newton, *Never at Rest*,

> *Newton was hardly an unknown man in philosophic circles before 1687. Nevertheless, nothing had prepared the world of natural philosophy for the Principia A turning point for Newton, who, after twenty years of abandoned investigations, had finally followed an undertaking to completion, the Principia also became a turning point for natural philosophy.*

The Principia became the new orthodoxy and reigned supreme for over 150 years. The first challenge to the Newtonian world view came from totally unexpected quarters: advances in the understanding of elecromagnetic phenomena. In the middle of the 19th century, a Scottish physicist James Clarke Maxwell achieved an astonishing synthesis of all the accumulated knowledge concerning these phenomena in just four vectorial equations. These equations further provided a specific value of the speed c of light. But no reference frame was specified. An absolute speed blatantly contradicted Galilean relativity, a cornerstone on which the Newtonian model of space-time rested. By then most physicists had developed deep trust in the Newtonian world and therefore concluded that Maxwell's equations can only hold in a specific reference frame, called the ether. But by doing so, they reverted back to the Aristotelian view that Nature specifies an absolute rest frame. A state of confusion remained for some 50 years.

It was the 26 year old Albert Einstein who grasped the true implications of this quandary: It was crying out, asking us to abolish Newton's absolute time. Einstein accepted the implications of Maxwell's equations at their face value and used simple thought experiments to argue that, since the speed c of light is a universal constant, the same for all inertial observers, the notion of absolute simultaneity is physically untenable. Spatially separated events which appear as simultaneous to one observer can not be so for another observer, moving uniformly with respect to the first. The Newtonian model of space-time can only

be an approximation that holds when speeds involved are all much smaller than c. A new, better model emerged and with it new kinematics, called special relativity. Time lost its absolute standing. Only the 4-dimensional space-time continuum had an absolute meaning. Space-time distances between events are well defined but time intervals or spatial distances between them depend on the state of motion of the observer, i.e., of the choice of a reference frame. The new paradigm came with dramatic predictions that were hard to swallow. Energy and mass lost their identity and could be transformed into one another, subject to the famous formula $E = Mc^2$. The energy contained in a gram of matter can therefore illuminate a town for a year. A twin who leaves her sister behind on earth and goes on a trip in a spaceship travelling at a speed near the speed of light for a year would return to find that her sister had aged several decades. So counter-intuitive were these implications that as late as the 1930s philosophers in prominent Western universities were arguing that special relativity could not possibly be viable. But they were all wrong. Nuclear reactors function on earth and stars shine in the heavens, converting mass in to energy, obeying $E = mc^2$. In high energy laboratories, particles are routinely accelerated to near light velocities and are known to live orders of magnitude longer than their twins at rest on earth.

In spite of these revolutions, one aspect of space-time remained Aristotelian: It continued to be a passive arena for all 'happenings', a canvas on which the dynamics of the universe are painted. In the middle of the 19th century, however, mathematicians discovered that Euclid's geometry that we all learned in school is only one of many possible geometries. This led to the idea, expounded most eloquently by Bernhard Riemann in 1854, that the geometry of physical space may not obey Euclid's axioms —it may be curved due to the presence of matter in the universe. It may not be passive but could act and be acted upon by matter. It took another 61 years for the idea to be realized in detail.

The grand event was Einstein's publication of his general theory of relativity in 1915. In this theory, space and time fuse to form a 4-dimensional continuum. The geometry of this continuum is *curved* and the amount of curvature in a region encodes the strength of the gravitational field there. Space-time is not an inert entity. It acts on matter and can be acted upon. As the American physicist John Wheeler puts it: *Matter tells space-time how to bend* and *space-time tells matter how to move*. There are no longer any spectators in the cosmic dance, nor a backdrop on which things happen. The stage itself joins the troupe of actors. This is a profound paradigm shift. Since all physical systems reside in space and time, this shift shook the very foundations of natural philosophy. It has taken decades for physicists to come to grips with the numerous ramifications of this shift and philosophers to come to terms with the new vision of reality that grew out of it. (For a detailed discussion, see [1].)

II.   GRAVITY IS GEOMETRY

It is as if a wall which separated us from the truth has collapsed. Wider expanses and greater depths are now exposed to the searching eye of knowledge, regions of which we had not even a pre-sentiment.

—Hermann Weyl. On General Relativity.

Einstein was motivated by two seemingly simple observations. First, as Galileo demonstrated through his famous experiments at the leaning tower of Pisa, the effect of gravity is universal: all bodies fall the same way if the only force on them is gravitational. Second,

gravity is *always* attractive. This is in striking contrast with, say, the electric force where unlike charges attract while like charges repel. As a result, while one can easily create regions in which the electric field vanishes, one can not build gravity shields. Thus, gravity is omnipresent and non-discriminating; it is everywhere and acts on everything the same way. These two facts make gravity unlike any other fundamental force and suggest that gravity is a manifestation of something deeper and universal. Since space-time is also omnipresent and the same for all physical systems, Einstein was led to regard gravity not as a force but a manifestation of space-time geometry. Space-time of general relativity is supple and can be visualized as a rubber sheet, bent by massive bodies. The sun for example, being heavy, bends space-time enormously. Planets like earth move in this curved geometry. In a precise mathematical sense, they follow the simplest trajectories called geodesics —generalizations of straight lines of the flat geometry of Euclid to the curved geometry of Riemann. So, when viewed from the curved space-time perspective, earth takes the straightest possible path. But since space-time itself is curved, the trajectory appears elliptical from the flat space perspective of Euclid and Newton.
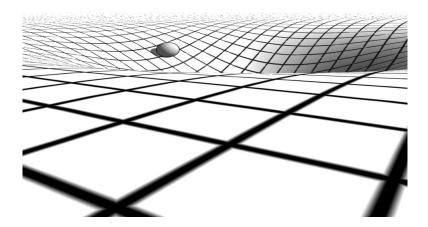

FIG. 1: An artist's depiction of planetary motion in general relativity. Heavy bodies such as our sun bend space-time. Planets go 'as straight as they can' in this curved geometry. In the flat space perspective of Newtonian physics, however, the same orbits appear to be elliptical, being eternally pulled to the sun by gravity. Thus the familiar gravitational force of Newtonian physics is just a 'poor man's way' of describing effects of space-time curvature using a flat space framework. Image: Boris Starosta, www. Starosta.com

The magic of general relativity is that, through elegant mathematics, it transforms these conceptually simple ideas into concrete equations and uses them to make astonishing predictions about the nature of physical reality. It predicts that clocks should tick faster in Kathmandu than in Bombay. Galactic nuclei should act as giant gravitational lenses and provide spectacular, multiple images of distant quasars. Two neutron stars orbiting around each other must lose energy through ripples in the curvature of space-time caused by their motion and spiral inward in an ever tightening embrace. Over the last thirty years, astute measurements have been performed to test if these and other even more exotic predictions are correct. Each time, general relativity has triumphed [2]. The accuracy of some of these observations exceeds that of the legendary tests of quantum electrodynamics. This combination of conceptual depth, mathematical elegance and observational successes is un-

precedented. This is why general relativity is widely regarded as the most sublime of all scientific creations [3].

III.   BIG BANG AND BLACK HOLES

The physicists succeeded magnificently, but in doing so, revealed the limitation of intuition, unaided by mathematics; an understanding of Nature, they discovered, comes hard. The cost of scientific advance is the humbling recognition that reality is not constructed to be easily grasped by the human mind.

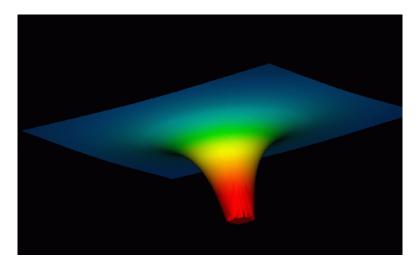—Edward O. Wilson. Consilience, The unity of Knowledge

FIG. 2:   A depiction of the universe originating at the big-bang and then expanding. Time runs vertically. In general relativity the curvature becomes infinite at the big-bang, tearing the very fabric of space-time continuum. The smooth conical surface depicts expanding space-time and the ragged edge at the bottom depicts the tearing of the fabric at the big-bang. Image: courtesy of Dr. Pablo Laguna.

General relativity ushered in the era of modern cosmology. At very large scales, the universe around us appears to be spatially homogeneous and isotropic. This is the grandest realization of the Copernican principle: our universe has no preferred place nor favored direction. Using Einstein's equations, in 1922 the Russian mathematician Alexander Friedmann showed that such a universe can not be static. It must expand or contract. In 1929 the American astronomer Edwin Hubble found that the universe is indeed expanding. This in turn implies that it must have had a beginning where the density of matter and curvature of space-time were infinite. This is the *big-bang*. Careful observations, particularly over the last decade, have shown that this event must have occurred some 14 billion years ago. Since then, galaxies are moving apart, the average matter content is becoming dilute. By combining our knowledge of general relativity with laboratory physics, we can make a number of detailed predictions. For instance, we can calculate the relative abundances of light chemical elements whose nuclei were created in the first three minutes after the big-bang; we can predict the existence and properties of a primal glow (the cosmic microwave background)

that was emitted when the universe was some 400,000 years old; and we can deduce that the first galaxies formed when the universe was a billion years old. An astonishing range of scales and variety of phenomena!

In addition, general relativity also changed the philosophical paradigm to phrase questions about the Beginning. Before 1915, one could argue –as Immanuel Kant did— that the universe could not have had a finite beginning. For, one could then ask, what was there before? This question pre-supposes that space and time existed forever and the universe refers only to matter. In general relativity, the question is meaningless: since space-time is now *born* with matter at the big-bang, the question "what was there *before*?" is no longer meaningful. In a precise sense, big-bang is a boundary, a frontier, where space-time ends. General relativity declares that physics stops there; it does not permit us to look beyond.

Through black holes, general relativity opened up other unforeseen vistas. The first black hole solution to Einstein's equation was discovered already in 1916 by the German astrophysicist Karl Schwarzschild, while he was serving on the front lines during the First World War. However, acceptance of its physical meaning came very slowly. A natural avenue for the formation of black holes is stellar collapse. While stars shine by burning their nuclear fuel, the outward radiation pressure can balance the inward gravitational pull. But after the fuel is all burned out, the only known force that can combat gravitational attraction comes from the quantum mechanical Pauli exclusion principle. During his celebrated voyage to Cambridge, the 20 year old Subrahmanyan Chandrasekhar combined principles of special relativity and quantum mechanics to show that, if a star is sufficiently massive, gravity would overwhelm the Pauli repulsive force. During the thirties, he refined his calculations, providing irrefutable arguments for the stellar collapse. However, the leading pre-eminent British astrophysicist of the time, Arthur Eddington, abhorred the idea of stellar collapse and declared that in the 'correct' calculation, special relativity had to be abandoned![1] This delayed not only the recognition of Chandrasekhar's work but also the general acceptance of black holes by several decades.

Ironically, even Einstein resisted black holes. As late as 1939, he published a paper in the Annals of Mathematics arguing that black holes could not be formed by the gravitational collapse of a star. The calculation is correct but the conclusion is an artifact of a non-realistic assumption. Just a few months later, American physicists Robert Oppenheimer and Hartland Snyder published their now classic paper establishing that black holes do in fact result. These are regions in which the space-time curvature is so strong that even light can not escape. Therefore, according to general relativity, to outside observers they appear pitch black. In the rubber sheet analogy, the bending of space-time is so extreme inside a black hole that space-time is *torn-apart,* forming a singularity. As at the big-bang, curvature becomes infinite. Space-time develops a final boundary and physics of general relativity simply stops.

And yet, black holes appear to be mundanely common in the universe. General relativity, combined with our knowledge of stellar evolution, predicts that there should be plenty of black holes with 10 to 50 solar masses, the end products of the lives of large stars. Indeed, black holes are *prominent* players in modern astronomy. They provide the powerful engines

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[1] Today, a Ph.D. student would fail his qualifying exam if were to make such an argument. Leading quantum physicists like Bohr and Dirac readily agreed with Chandrasekhar privately, but did not think it was worthwhile to point out Eddington's error publicly. It was only in 1983 that Chandrasekhar was awarded the Nobel prize for this seminal discovery. For details, see, e.g., [4].

for the most energetic phenomena in the universe such as the celebrated gamma ray bursts in which an explosion spews out, in a few blinding seconds, as much energy as a 1000 suns do in *their entire lifetime.* One such burst is seen every day. Centers of all elliptical galaxies appear to contain huge black holes of millions of solar masses. Our own galaxy, the Milky Way, has a black hole of about 3.2 million solar masses at its center.

IV.   BEYOND EINSTEIN

> *A really new field of experience will always lead to crystallization of a new system of scientific concepts and laws.When faced with essentially new intellectual challenges, we continually follow the example of Columbus who possessed the courage to leave the known world in an almost insane hope of finding land again beyond the sea.*

—W. Heisenberg. Recent Changes in the Foundation of Exact Science

General relativity is the best theory of gravitation and space-time structure we have today. It can account for a truly impressive array of phenomena [1, 2] ranging from the grand cosmic expansion to the functioning of a mundane global positioning system on earth. But it is incomplete because it ignores quantum effects that govern the sub-atomic world. Moreover, the two theories are dramatically different. The world of general relativity has geometric precision, it is deterministic; the world of quantum physics is dictated by fundamental uncertainties, it is probabilistic. Physicists maintain a happy, schizophrenic attitude, using general relativity to describe the large scale phenomena of astronomy and cosmology and quantum mechanics to account for properties of atoms and elementary particles. This is a viable strategy because the two worlds rarely meet. Nonetheless, from a conceptual standpoint, this is highly unsatisfactory. Everything in our experience as physicists tells us that there should be a grander, more complete theory from which general relativity and quantum physics arise as special, limiting cases. This would be the quantum theory of gravity. It would take us beyond Einstein.[2]

At the big-bang and black hole singularities the world of the very large and of the very small meet. Therefore, although they seem arcane notions at first, these singularities are our gates to go beyond general relativity. It is now widely believed that real physics can not stop there. Rather, general relativity fails. We need to dramatically revise, once again, our notions of space and time. We need a new syntax.

Creation of this syntax is widely regarded as the greatest and the most fascinating challenge faced by fundamental physics today. There are several approaches. While they generally agree on a broad list of goals, each focuses on one or two features as the central ones, to

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[2] Contrary to the common belief —rooted in Einstein's later views on incompleteness of quantum mechanics— he was quite aware of this limitation of general relativity. Remarkably, he pointed out the necessity of a quantum theory of gravity already in 1916! In a paper in the *Preussische Akademie Sitzungsberichte* he wrote: *"Nevertheless, due to the inneratomic movement of electrons, atoms would have to radiate not only electromagnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in Nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics but also the new theory of gravitation."*

be resolved first, in the hope that the other problems 'would take care of themselves' once the 'core' is well-understood. Here, I will focus on *loop quantum gravity* which originated in our group some twenty years ago and has been developed by about two dozen groups world wide [5]. It is widely regarded as one of the two leading approaches, the other being string theory [7].

In general relativity, space-time is modelled by a continuum. The new idea is that this is only an approximation, which would break down at the so called Planck scale —the unique length, $\ell_{\rm pl} = \sqrt{G\hbar/c^3} \sim 10^{-33}$ cm, that can be constructed from Newton's constant of Gravitation G, Planck's constant $\hbar$ of quantum physics and the speed of light c. This scale is truly minute, some 20 orders of magnitude smaller than the radius of a proton.[3] Therefore, even in the highest energy particle accelerators on earth, one can safely work with a continuum. But the approximation *would* break down in more extreme situations, in particular, near the big-bang and inside black holes. There, one must use a *quantum space-time* of loop quantum gravity.

What is quantum space-time? Look at the sheet of paper in front of you. For all practical purposes, it seems continuous. Yet we know that it is made of atoms. It has a discrete structure which becomes manifest only if you zero-in using, say, an electron microscope. Now, Einstein taught us that geometry is also a physical entity, on par with matter. Therefore, it should also have an atomic structure. To unravel it, in the mid 90's researchers combined the principles of general relativity with quantum physics to develop a *quantum theory of geometry*. Just as differential geometry provides the mathematical language to formulate and analyze general relativity, quantum geometry provides the mathematical tools and physical concepts to describe quantum space-times [5, 6].

In quantum geometry, the primary objects —the fundamental excitations of geometry— are one dimensional. Just as a piece of cloth appears to be a smooth, two dimensional continuum although it is obviously woven by one dimensional threads, physical space appears as a three dimensional continuum, although it is in fact a coherent superposition of these one dimensional excitations. Intuitively, then, these fundamental excitations can be thought of as *quantum threads* which can be woven to create the fabric of space-time. What happens, then, near space-time singularities? There, the continuum approximation fails. The quantum fluctuations are so huge that quantum threads can no longer be frozen into a coherent superposition. The fabric of space-time is ruptured. Continuum physics rooted in this fabric stops. But the quantum threads are still meaningful. Using a quantum generalization of Einstein's equations one can still do physics, describe what happens in the quantum world. In the absence of a space-time continuum, many of the notions habitually used in physics are no longer available. New concepts have to be introduced, new physical intuition has to be honed. In this adventure, quantum Einstein's equations pave the way.

Using these equations recently the big-bang has been analyzed in some detail (see, e.g., [6, 8]). It turns out that the partial differential equations of Einstein's, adapted to the continuum, have to be replaced by difference equations, adapted to the discrete structures of quantum geometry. Except very near the big-bang, equations of general relativity provide an excellent approximation to the more fundamental ones. However, the approximation breaks down completely near the big-bang, when the density ρ of matter approaches the Planck density $\rho_{\rm pl} = c^3/G^2\hbar \approx 10^{94}$gm/cc. In quantum geometry, space-time curvature

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[3] For non-experts, it is often difficult to imagine how large a number 10^{20} is. So, the following illustration may help: $\$10^{20}$ would suffice to cover the US budget for a 100 million years at the 2005 rate!

does become very large in this *Planck regime*, but not infinite. Very surprisingly, quantum geometry effects give rise to a new *repulsive* force, which is so strong that it overwhelms the usual gravitational attraction. General relativity breaks down. The universe bounces back. But quantum Einstein's equations enable us to evolve the quantum state of geometry and matter right through this Planck regime. The big bang is replaced by a quantum bounce.
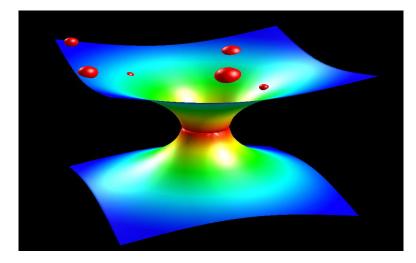

FIG. 3: An artist's representation of the extended space-time of loop quantum cosmology. Time again runs vertically. General relativity provides only the top half of this space-time which originates in the big-bang (see figure 2). Quantum Einstein's equations extend this space-time to the past of the big-bang. The pre-big-bang branch is contracting and the current post-big-bang branch is expanding. The band in the middle represents the 'quantum bridge' which joins the two branches and provides a deterministic evolution across the 'deep Planck regime'. Image: courtesy of Dr. Cliff Pickover, www.pickover.com

Reliable numerical calculations have been performed in the strict spatially homogeneous isotropic case. Continuum turns out to be a good approximation outside the Planck regime also on the 'other side of the big-bang' [6, 8]. More precisely, in a forward-in-time motion picture of the universe, there is a contracting pre-big-bang branch well described by general relativity. However, when the matter density is approximately $0.8\rho_{\rm pl}$, the repulsive force of quantum geometry, which is negligible until then, now becomes dominant. Instead of continuing the contraction into a big-crunch, the universe undergoes a big bounce, joining on to the post-big-bang expanding branch we now live in. Classical general relativity describes both branches very well, except in the deep Planck regime. There the two branches are joined by a quantum bridge, governed by quantum geometry.

The emergence of a new repulsive, quantum force has a curious similarity with the repulsive force in the stellar collapse discussed in section III. There, a repulsive force comes into play when the core approaches a critical density, $\rho_{\rm crit} \approx 6 \times 10^{16}$gms/cc, and can halt further collapse, leading to stable neutron stars. This force, with its origin in the Pauli exclusion principle, is *associated with the quantum nature of matter*. However, as indicated in section III, if the total mass of the star is larger than, say, 5 solar masses, classical gravity overwhelms this force. The *quantum geometry repulsive force* comes into play at *much* higher densities. But then it overwhelms the standard gravitational attraction, *irrespective of how massive the collapsing body is.* Indeed, the body could be the whole universe! The perspective of loop quantum gravity is that it is this effect that prevents the formation of singularities in the real world, extending the 'life' of space-time through a quantum bridge.

Currently, work is under way to extend these results to more and more sophisticated models which incorporate inhomogeneities of the present day universe. If the above scenario turns out to be robust, there will be fascinating philosophical implications for the issue of the Beginning and the End. For, the very paradigm to pose questions will again be shifted. If the questions refer to the notion of time that Einstein gave us, there was indeed a Beginning. Not at the big-bang though, but 'a little later' when space-time can be modelled as a continuum. But if by Beginning one means a firm boundary beyond which physical predictions are impossible, then the answer is very different from that given by general relativity: in the more complete theory, there is no such Beginning.

To summarize then, thanks to Einstein, our understanding of space and time underwent a dramatic revision in the 20th century. Geometry suddenly became a physical entity, like matter. This opened-up entirely new vistas in cosmology and astronomy. But a new paradigm shift awaits us again in the 21st century. Thanks to quantum geometry, the big-bang and black hole singularities are no longer final frontiers. The physical, *quantum* space-time is much larger than what general relativity had us believe. The existence of these new and potentially vast unforeseen domains has already provided a fresh avenue to resolve several long standing, problems concerning both cosmology and black holes in fundamental physics. Even more exciting opportunities arise from new questions and the rich possibilities that this extension opens up.

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